RYERSON UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE

CPS 616
FINAL EXAM
WINTER 2015

NAME: _____

STUDENT ID: _____

INSTRUCTIONS

- This exam is 3 hours long.
- This exam is out of 70 and is worth 35% of the course mark.
- This is a closed book exam. However, one double-sided letter-sized crib sheet is allowed.
- This exam is single-sided and has 6 pages including this front page.
- The first part of this exam is multiple choice. Please enter your answers to this part on the bubble sheet provided.
- The second part of this exam is short answer. Please answer all questions of this part directly on this exam.

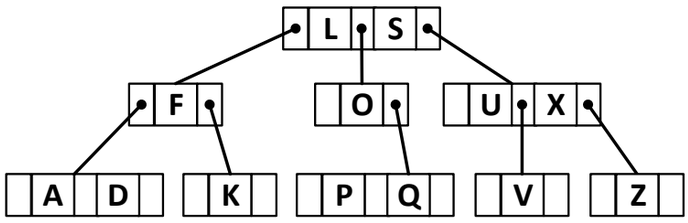
PART 1 - MULTIPLE CHOICE

Instructions

Please enter your answers on the bubble sheet with your name.
Use pencil only.
You may guess. Marks will not be deducted for false answers..

Questions (15 marks)

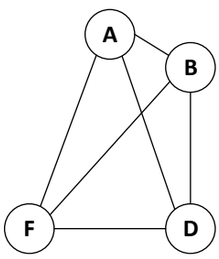
1. (2 marks) Is this graph:



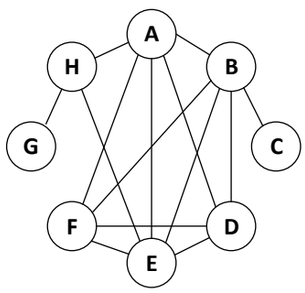
A. An AVL Tree
B. A 2-3 Tree
C. A Red-Black Tree
D. None of the above

2. (2 marks) Given the following graphs, which statement is correct:

Graph G



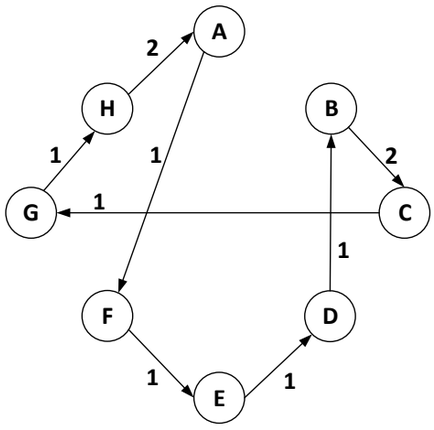
Graph H



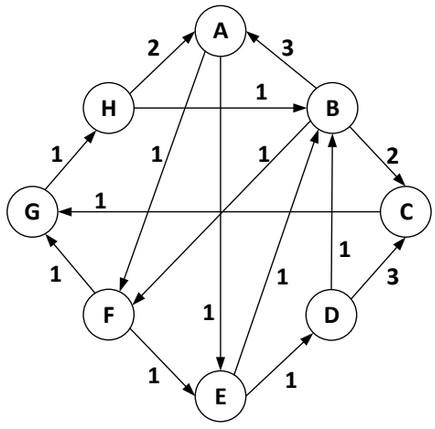
A. G is the complement of H
B. G is a non-maximal clique of H
C. G is a maximal clique of H
D. G is a spanning tree of H
E. None of the above

3. (3 marks) Given the following graphs, which statement is correct:

Graph F



Graph G



A. F is an Eulerian circuit of G which **is not** a solution to the Travelling Salesman problem for G
B. F is a Hamiltonian circuit of G which **is not** a solution to the Travelling Salesman problem for G
C. F is an Eulerian circuit of G which **is** a solution to the Travelling Salesman problem for G
D. F is a Hamiltonian circuit of G which **is** a solution to the Travelling Salesman problem for G
E. None of the above

The next 2 questions refer to the following linear programming problem whose feasible region and solution is graphed below. **The two questions are independent of each other.**

Linear Programming problem

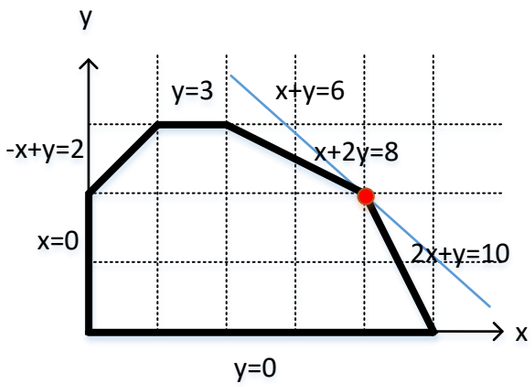
Maximise

$x+y$

For

$0 \le x$
$0 \le y \le 3$
$-x+y \le 2$
$x+2y \le 8$
$2x+y \le 10$

Feasible region and solution



4. (2 marks) What happen to this problem if you change the constraint $y \le 3$ to be $y \ge 3$?

   A. The problem becomes unfeasible and has no solution
   B. The problem becomes unbounded and has no solution
   C. The problem remains feasible and has a single optimal solution
   D. The problem remains feasible but has more than one optimal solution

5. (2 marks) What happen to this problem if you change the constraint $y \le 3$ to be $y \ge 5$?

   A. The problem becomes unfeasible and has no solution
   B. The problem becomes unbounded and has no solution
   C. The problem remains feasible and has a single optimal solution
   D. The problem remains feasible but has more than one optimal solution

6. (2 marks) Which of the following statements is **not true** about the following problem P:
   find the maximum element of an n×n array

   A. $\Omega(n)$ is a lower bound for P
   B. $\Omega(n)$ is a tight lower bound for P
   C. $\Omega(n^2)$ is a lower bound for P
   D. $\Omega(n^2)$ is a tight lower bound for P

7. (2 marks) It can be proven that for two positive integers m and n such that $m \le n$ the worst case cost of the Euclidian algorithm to calculate gcd(m,n) is $O(\log_{10} m)$.

   Also, as you know, lcm(m,n) = m.n / gcd(m,n)

   What can we therefore conclude about the cost C of calculating lcm(m,n) using the Euclidian algorithm?

   A. $C \in O(\log_{10} m)$
   B. $C \in O(1/\log_{10} m)$
   C. $C \in O(n/\log_{10} m)$
   D. $C \in O(n.m/\log_{10} m)$

PART 2 - SHORT ANSWERS - PLEASE WRITE YOUR ANSWERS DIRECTLY IN THIS EXAM

8. (20 marks) - Huffman Code

- (16 marks) Construct a Huffman tree and corresponding Huffman code for the following data. Your internal nodes should contain total frequencies, and your branches should be labelled with 0s and 1s

Symbol	' '	a	b	d	e	g	o	y
Frequency	0.20	0.15	0.11	0.13	0.17	0.10	0.10	0.04
Codeword								

When there are ties for the selection of the subtrees that will form a tree, they are broken in accordance with the rules below in the following order:

1) The shallowest subtree(s) is(are) selected
2) The subtree(s) containing the earliest element in alphabetical order is(are) selected

When building a tree from 2 subtrees, the left and right subtrees are placed in accordance with the rules below in the following order:

1) The lighter subtree is on the left
2) The shallowest subtree is on the left
3) The tree containing the earliest element in alphabetical order is on the left

(2 marks) Encode the string "**doggy bag**" (2 words with a blank between them) using the Huffman code you designed in a)

(2 marks) Decode the word 10110100101000111010111 using the Huffman code you designed in a)

9. (20 marks) Non-Deterministic Turing Machines (NTMs)

Here is a NTM with 3 tapes called T1,T2,T3.
The symbol "-" is the blank symbol.
The actions for each tape have the format: <new symbol> , <move>
where the possible moves are: R=move right, L=move left, S=stay

State	Current Symbol T1	T2	T3	Action T1	T2	T3	New State	Explanation
q_1	1	-	-	1,R	1,R	-,S	q_2	
	1	-	-	1,R	-,S	1,R	q_3	
q_2	0	-	-	0,R	0,R	-,S	q_2	
	1	-	-	1,S	-,S	-,S	q_1	
	-	-	-	-,S	-,L	-,L	q_4	
q_3	0	-	-	0,R	-,S	0,R	q_3	
	1	-	-	1,S	-,S	-,S	q_1	
	-	-	-	-,S	-,L	-,L	q_4	
q_4	-	0	0	-,S	0,L	0,L	q_4	
	-	0	1	-,S	0,S	1,L	q_4	
	-	1	0	-,S	1,L	0,S	q_4	
	-	1	1	-,S	1,L	1,L	q_4	
	-	-	-	-,S	-,S	-,S	q_5	
q_5								Accept

- (1 mark) Circle all the non-deterministic states in the first column of the table

(6 marks) Explain in English in the last column of the table what the states q_1 to q_4 do.

(3 marks) Why does state q_5 accept the input on T1? In other words, what does this NTM do?

(2 marks) If this NTM accepts an input string on T1 which has **n** 1's, how many times will it have read a 1 before it accepted the string? The answer should be a function of **n**. In this question "reading a 1" means that one of the tape heads was over a 1 and moved left or right away from it.

(3 marks) Why is the previous question asking you to count how many times a 1 is read and not to count how many times a 0 is read?

(3 marks) Is the problem that this NTM solves in NP class? Explain your answer.

(2 marks) Is the problem that this NTM solves in P class? Explain your answer.

10. (15 marks) Dynamic Programming

Here is pseudocode for a recursive program

```
// This function calculates x to the power of n
//  You can assume that x is positive and n is a non-negative integer
exponent (x, n)
        if n=0 return 1
        if n=1 return x
        return exponent(x, n div 2) * exponent(x, n - (n div 2))
```

- (2 marks) Assuming that the multiplication in the last line is the basic operation in this algorithm, what is the exact cost of this algorithm as a function of n?

(8 marks) Modify this pseudocode so that it solves exactly the same problem by recursing exactly the same way, but using a dynamic programming approach, i.e:

- "exponent(x, n div 2) * exponent(x, n - (n div 2))" will still be part of your code
- solutions to subproblems are stored the first time they are calculated and never calculated again.

You can use the following global array:

```
global exp[0,..,n]          // to store intermediate results
                            // This array is initialized with 0s
```

- (2 marks) What is the best case cost of your new algorithm as a function of n?

- (3 marks) Give an upper bound for the worst case cost of your new algorithm as a function of n. (Hint: look at n=127)
  Do not spend too much time on this question, it is only worth 3 marks!